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Synchronization induced by disorder of phase directions

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We study the influence of randomly distributed phase directions of external force in an array of coupled pendula, instead of studying the influence of continuous phase. We find that with the increase of the absolute value of the phase, the chaotic behaviors of the coupled arrays may be controlled and different synchronized patterns can be induced. These results demonstrate that by introducing the randomness of the phase directions, rather than the continuous value of the phase, it can lead to a synchronization in nonlinear systems. This finding may provide a new insight for understanding the mechanism of disorder induced synchronization.

Keywords: Phase direction; random distribution; synchronization; nonlinear system.

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1. Introduction

Physical and biological systems have been widely modeled by networks with coupled nonlinear oscillators.[1] Examples cover nervous system,[2,3] Josephson junctions[4,5] and solid state lasers.[6] Among these researches, one of the significant issues is to control the chaotic behavior of the systems[7,8] and a variety of methods have been proposed both for dissipative systems[9–19] and conservative systems.[20–22] An interesting finding is that the disorder, which is intuitively regarded as a destructive power for regular dynamics, can also be used to control chaos.[23–28] For example, with a certain amount

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of disorder to the time delays of the interactions among an array of pendula, the chaotic behavior of the array can be tamed to periodic behaviors.[29]

In general, dynamic systems can be classified into two classes: continuous systems and discrete systems.[30] Synchronization induced by the stochastic initial phase of the external forces has been well-studied in the continuous systems and several approaches have been presented to define the phase of flow.[7] It is found that by randomizing the initial continuous phase of the external forces of a group of coupled oscillators, the chaotic behaviors of the oscillator can be controled.[31] Specifically, when the range of randomized phases is sufficiently large, the behaviors of the oscillators can be synchronized and different synchronized patterns emerge for different ranges. As we know, in these continuous systems, there is a trajectory direction and its rotation shows the variation of phase, i.e. phase represents the rotation of trajectory.[32] However, we do not have the similar concept for the discrete systems. The reason is that discrete systems can be considered as the behavior of flow on a Poincaré section, in this sense there is no trajectory direction for a discrete system and thus we cannot define the phase for it. Therefore, for two coupled identical chaotic maps, what we can do is to study their complete synchronization but not the phase synchronization. When the coupling strength is not large enough to induce the complete synchronization, we will be in an awkward position to describe the correlated behaviors of coupled maps. For example, the variables of two coupled Logistic maps may show a consistent increase or decrease behavior for a finite coupling strength, which cannot be described by the concept of complete synchronization as their values are always different. For describing this kind of consistence, Wang *et al.* introduced a concept of "phase direction" to study relative dynamics. Ho *et al.* investigated the phase synchronization of inhomogeneous globally coupled map lattices.[34] After that, the phase direction has been widely used to characterize the discrete systems due to its convenient application, such as the intermittent phase synchronization of discrete systems,[35] the behavior of phase direction at crises,[36] and other coupled discrete dynamics on a variety of networks.[37–40]

However, to the best of our knowledge, there is no relative work to investigate the influence of the random phase direction of external forces on the chaotic systems. In this paper, we study an array of coupled pendula with random initial phase direction of external forces. Specifically, distinct from the continuous phase value, here the absolute value of phases of all the oscillators are the same while the value could be either positive or negative denoting the direction of the phase. Our main results show: (1) the disorder of phase direction could control the chaotic behavior of the system; and (2) different synchronized patterns emerge at different disorder extents.

2. Random Phase Direction Model

We here consider a Frenkel–Kontorova chain which is composed of an array of nonlinear pendula subjected to an external force and damped environment. The motion of the one-dimensional (1D) array (chain) is described by the following equation

$$ml^2\,\ddot{\theta}_n + \gamma\dot{\theta}_n = -mgl\sin\theta_n + \tau' + \tau\sin(\omega t + A_n\varphi) + s[\theta_{n+1} + \theta_{n-1} - 2\theta_n], \quad n = 1, 2, \ldots, N, \tag{1}$$

where the parameters are taken as follows: the mass of the oscillator $m = 1$, the length $l = 1$, the acceleration due to gravity $g = 1$, the damping $\gamma = 0.75$, the dc torque $\tau' = 0.7155$, the ac torque $\tau = 0.4$, the angular frequency $\omega = 0.25$, the coupling strength $s = 0.5$. Here, A_n can be either $+1$ or -1 with equal probability. In other words, the value of A_n is randomly chosen from the set $\{+1, -1\}$ and for each element the possibility equals 0.5. The value $\varphi \in [0, \pi/2]$ denotes the absolute value of phases, and all pendula have the same value of φ. Hence, a larger φ corresponds to a larger extent of disorder. In this paper, we employ the periodic boundary condition, i.e. $\theta_0 = \theta_N$, $\theta_{N+1} = \theta_1$. Moreover, we fix $N = 50$ through the simulation, if not otherwise specified.

3. Simulations and Results

To facilitate our observation of how the value φ influences the global spatiotemporal behavior of the system, we consider the average velocity

$$\sigma(jT) = \frac{1}{N}\sum_{n=1}^{N}\dot{\theta}_n(jT), \tag{2}$$

at times that are integer multiples of the forcing period $T = 2\pi/\omega$. We find that the oscillators process periodic behavior when φ is sufficiently large.

Figure 1 shows how σ changes with φ for $N = 50$, where σ is measured at $t = 60T, 61T, \ldots, 80T$ for each φ. It is easy to see that when $0 \leq \varphi < 0.05$, σ is diverse from 0.05 to 0.45, which appears chaotic behaviors. When φ is in a medium range around $[0.05, 0.11]$, it shows $2T$ attractors. After that, σ is multi-valued when $0.12 \leq \varphi < 0.14$. When $\varphi \geq 0.15$, σ moves into a $1T$ attractor.

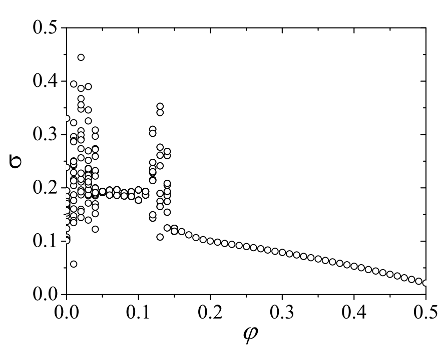
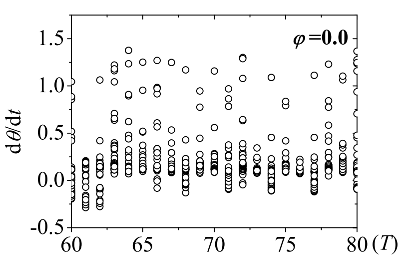


Fig. 1. How the average velocity σ depends on the absolute value of phase φ for $N = 50$, where σ is taken at $t = 60T, 61T, \ldots, 80T$ for each φ.

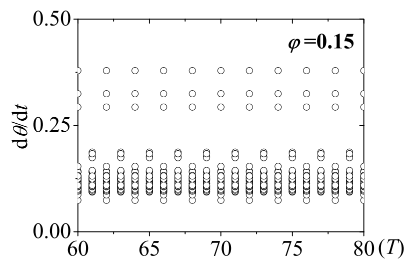
How do the collective behaviors in Fig. 1 show up? To understand how the system evolves from chaotic behavior to periodic ones, we present a detailed plot of the evolution of each individual oscillator from different φ value. Figure 2 shows three typical evolution for the cases of $\varphi = 0, 0.15, 0.3$, respectively. Obviously, Figs. 2(a)–2(c) present chaotic, $2T$ and $1T$ attractors, respectively. Their averages give just the values observed in Fig. 1. This figure shows that the average velocity σ has the same type of attractors as all the oscillators for different φ.

Furthermore, from Figs. 1 and 2, we can see that with an increase φ there could be more periodic attractors. Despite the observation that the system eventually moves to a $1T$ attractor when φ is large enough, it is interesting to observe what type of attractors happen when the value of φ is at a medium level. For doing so, we calculate the possibility of the system to reach a $1T$, $2T$ and chaotic attractors over abundance of different realizations. Figure 3 shows the possibility of these attractors as a function of φ. We can see that when φ is smaller than 0.05, the probability of "chaos" keeps 1, while the probabilities of $1T$ and $2T$ is 0, which means the system behavior is mainly chaotic. While when φ is around $[0.05, 0.11]$, $1T$ and $2T$ attractors emerge, which means other type of periodic patterns are more likely to show up. However when P is larger than 0.12, $1T$ attractors dominates the system behavior.

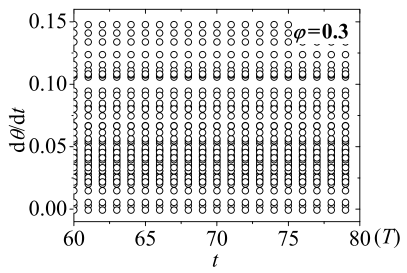
With the increase of φ, except for the system favoring more to the periodic behavior, it is also observed that the oscillators may become largely synchronous at a medium level of φ. For measuring how the disorder parameter φ influences the



(a)



(b)



(c)

Fig. 2. Typical evolutions of individual oscillators for $N = 50$ where (a) to (c) represent the cases of $\varphi = 0.0, 0.15$ and 0.3, respectively.

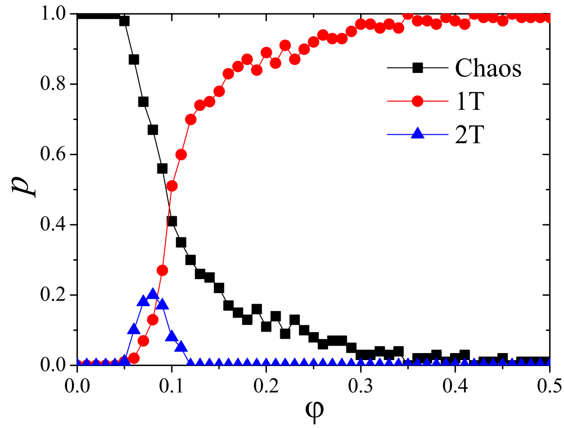


Fig. 3. (Color online) Probability P of chaotic dynamics and several regular behaviors versus the absolute value of phase φ in a chain of $N = 50$ coupled oscillators. P is determined by averaging over 200 different sets of realizations.

synchronization, we introduce the self-correlation and the cross-correlation. The self-correlation is defined as follows

$$D = \frac{1}{N}\sum_i D_i, \tag{3}$$

with

$$D_i = \frac{\int_{T_0}^{T_0+T} dt\dot{\theta}_i(t)\dot{\theta}_i(t+T)}{[\int_{T_0}^{T_0+T} dt\dot{\theta}_i^2(t)\int_{T_0}^{T_0+T} dt\dot{\theta}_i^2(t+T)]^{1/2}}, \tag{4}$$

where T_0 is a time after the transient process. The "hollow circles" in Fig. 4 shows how D changes with φ. It is easy to see that D does not change a lot when $\varphi < 0.05$,

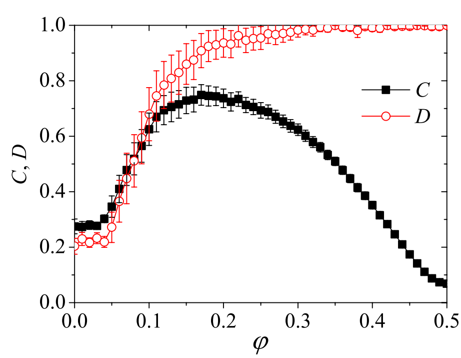


Fig. 4. (Color online) Correlations D and C versus φ, where the "hollow circles" and "solid squares" represent the self-correlation and cross-correlation for $N = 50$, respectively. The average is performed over 200 different sets of the realizations and the error bars show the standard deviation.

which means a small amount of disorder does not have much influence on D. However, when φ is larger than 0.05, D increases monotonously with φ until $D = 1$, which means the periodic patterns appear are gradually transformed into low-periodic patterns and eventually reach the $1T$ attractor. When $\varphi > 0.35$, D remains which indicates that the pattern become strictly $1T$ attractor.

Now, we study the mutual synchronization among different oscillators. For measuring this relationship, the cross-correlation is defined as

$$C = \frac{2}{N(N-1)} \sum_{i<j} C_{ij}, \tag{5}$$

with

$$C_{ij} = \frac{\int_{T_0}^{T_0+T} dt \dot{\theta}_i(t) \dot{\theta}_j(t)}{[\int_{T_0}^{T_0+T} dt \dot{\theta}_i^2(t) \int_{T_0}^{T_0+T} dt \dot{\theta}_j^2(t)]^{1/2}}, \tag{6}$$

where C_{ij} denotes the correlation between the ith and jth oscillators. The "solid squares" in Fig. 4 shows C as a function of φ. When the disorder parameter is small, the disorder of phase direction results in more synchronized oscillators of the array. And at a mediate value of $\varphi = 0.17$, synchronization in the array reaches a peak value. However, further increase the disorder parameter results in less synchronization of the oscillators as a distinction of the self-correlation function D, which means the disorder of the phase direction does not favor the synchronization when the disorder is too large.

It is interesting to study how the self-correlation and cross-correlation depends on the number of oscillators. By increasing N, we find that the phase transition becomes more clear; however, when N increases further, such as $N > 500$, we observe that for larger φ, the self-correlation D will stay around unity, but the cross-correlation C will decrease. Thus, we infer that in the thermodynamic limit, the transition to unity will be kept for the curve D, but disappear for the curve C. As D measures the self-correlation, it does not depend on concrete periodic patterns or chaos. However, D will be influenced by these multiple attractors. For example, when the oscillator i is in period 1 and the oscillator j is in period 4, their different periods will make it difficult for C to be close to unity, resulting in a difference between D and C.

4. Discussions and Conclusions

In the communication of real physical or biological systems, phase directions in signals are more universal than continuous phase. For example, in the complex traffic systems, the driving behaviors of the cars are chaotic. Our results suggest that by adding several disorder cars the traffic flow may result in synchronization because the added cars can restrict the chaotic cars following some patterns. And therefore our work can benefit to solve the serious traffic jam problem. This finding may provide further insight into traffic control and management systems.

In conclusions, we have uncovered a phenomenon of controling chaotic dynamics with disordered phase directions. This approach is equivalent to the approach of distributed random phases. When the disorder parameter φ is significant, the disordered phase direction can tame the chaotic behavior of the system to periodic patterns. Moreover, the oscillators can become largely synchronized at a mediate level of φ.

Acknowledgments

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