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Local-world and cluster-growing weighted networks with controllable clustering

Chun-Xia Yang*, Min-Xuan Tang†, Hai-Qiang Tang‡
and Qiang-Qiang Deng§

School of Information and Control
Nanjing University of Information Science and Technology
Nanjing 210044, P. R. China
*y.cx@163.com
†tangminxuan@qq.com
‡715618293@qq.com
§wangj0809@163.com

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We constructed an improved weighted network model by introducing local-world selection mechanism and triangle coupling mechanism based on the traditional BBV model. The model gives power-law distributions of degree, strength and edge weight and presents the linear relationship both between the degree and strength and between the degree and the clustering coefficient. Particularly, the model is equipped with an ability to accelerate the speed increase of strength exceeding that of degree. Besides, the model is more sound and efficient in tuning clustering coefficient than the original BBV model. Finally, based on our improved model, we analyze the virus spread process and find that reducing the size of local-world has a great inhibited effect on virus spread.

Keywords: Complex networks; local-world; power-law distribution; tunable clustering coefficient.

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1. Introduction

During recent years, we have witnessed a soaring movement of research in complex networks.[1–3] As before, many scholars have still devoted to answer the following questions: what is the connection between topological characteristics of one network and its unique dynamical properties? What is the origin why a special topology is chosen by a given real network? For the latter, Barabsi–Albert (BA) scale-free network model introduces growth and preferential attachment mechanism to mimic many complex real-world networks such as World Wide Web, Internet, cellular networks, ecological networks, citation networks, movie actor collaboration networks and so on.[4] Here, BA model only presents connections between nodes, cannot describe the strength of interaction between them. Then, taking the unique interaction between any two individuals in real-world into account and introducing weight-driven dynamics and a weight reinforcement mechanism coupled to the local network growth, Barrat, Barthelemy and Vespignani built a famous model (BBV model) in 2004[5] and raised a hot wave of studying weighted network evolution model. Henceforth, many scholars improved the BBV model to make it yield the evolution law of real network system. For example, Wang *et al.* proposed a traffic-driven evolution model of weighted technological networks,[6] where the traffic and topology interact mutually. Adding a restricted condition that node strength does not exceed a certain value, Liu *et al.* proposed a new weighted evolving network model (LBBV model).[7] Additionally, some models have been constructed by incorporating triangle coupling mechanism like Wang's weighted scale-free network model which studied the traffic and routing of networks,[8] the model of Kumpula *et al.* which provided a starting point for understanding the formation of communities in weighted networks, and so on.[9] Other models absorbed local-world selection mechanism like the model of Xuan *et al.* which expressed important properties of the supply chain system,[10] the model of Wen *et al.* which characterized the Internet topology,[11] and so on. It is worth noting that triangle coupling and local-world selection often exist together in real-life systems, but it has been ignored in the above works. So motivated by such ideas, based on the BBV model, we constructed an improved weighted network model (IBBV model) which incorporates the above two mechanisms. Fortunately, our model could generate many features confirmed in large numbers of real networks. Furthermore, based on our improved model, the virus spread process is investigated and the fact that the infection density and the virus spread rate will increase with the increasing of local-world size could be found.

The remainder of this paper is organized as follows: Section 2 provides the model. Section 3 presents theoretical analysis of our model. Section 4 reports simulation results of the model. Section 5 analyze the virus spread behavior on the model. In the end, concluding remarks are stated in Sec. 6.

2. The Model

Here, based on the previous work, considering both triangle coupling mechanism and local-world selection mechanism, we established a network model called improved BBV. For the sake of simplicity, the network is an undirected network. The algorithm is as follows:

(i) Initialization. The network begins with an initial connected network of m_0 nodes, all the first value of link weight is set to be $w_0 = 1$. Given a constant $\delta > 0$ and probabilities $p \in [0, 1]$ and $q \in [0, 1]$.
(ii) Topological growth. New nodes are added to the network one at a time. At each time-step, a new node n with m edges is added. After t time-steps, the network has

$N \doteq t + m_0$ nodes. When the new node obtains global network information at probability p, it will preferentially attach to an existing node i in the global world at a probability:

$$\prod_{\text{Global}} s_i = \frac{s_i}{\sum_{j\in\text{Global}} s_j}, \tag{1}$$

where, s_i is the strength of node i, $s_i = \sum_{j\in\Gamma(i)} w_{ij}$, $\Gamma(i)$ is a set of neighbors of node i. If the new node only gets local information at probability $1-p$, then there randomly exists a local-world with $M(M \leq N)$ nodes, the new node will preferentially attach to an existing node i in the local-world at a probability:

$$\prod_{\text{Local}} s_i = \Pi(i \in \text{Local}) \frac{s_i}{\sum_{j\in\text{Local}} s_j}, \tag{2}$$

where $\prod(i \in \text{Local}) = \frac{M}{m_0+t}$. For the m edges of a new node n, the first link is established by the above preferential attachment mechanism at a probability $1-q$, the other $m-1$ links are constructed by the following method: select $m-1$ neighbor nodes of the chosen node i at a probability q, and each node among them is connected with the new node n at a probability:

$$\prod_{j\to n}(j \in \Gamma(i)) = w_{ij}/s_i. \tag{3}$$

Therefore, $n = (m-1)q$ times of triangle coupling are needed once a new node is added. At this time if all the neighbors of the chosen node i has already been connected with the new node n, then do a preferential attachment instead at the probability shown in Eq. (1) or Eq. (2).
(iii) Weights dynamics. The weight of each new edge (n, i) is initially set to be $w_0 = 1$. The creation of this edge will introduce variations of the traffic across the network, which will induce a total increase δ of the total traffic. For the sake of simplicity, the case where the addition of a new edge on node i only trigger local rearrangements of weights among the edges and where the traffic perturbation is proportionally distributed among the edges on the existing neighbors $j \in \Gamma(i)$ is considered. The rule is as follows:

$$w_{ij} \to w_{ij} + \Delta w_{ij}, \tag{4}$$

$$\Delta w_{ij} = \delta \frac{w_{ij}}{s_i}. \tag{5}$$

This rule yields a total strength increase for node i of $\delta + w_0$, implying that $s_i \to s_i + \delta + w_0$. After the weights have been updated, the growth process is iterated by introducing a new vertex, i.e. going back to step (ii) until the desired size of the network is reached.

3. Theoretical Analysis

When a new node n enters the network, the strength s_i of a node i could be affected by the following four factors: (i) It is preferentially selected and connected with the node n; (ii) It is connected to the node n by the triangle coupling mechanism; (iii) One of its neighbors $j \in \Gamma(i)$ is preferentially selected and connected with the node n; (iv) One of its neighbors $j \in \Gamma(i)$ is connected to the node n due to triangle coupling mechanism. Thus, s_i changes over time as shown below:

(i) When the new node obtains global network information, s_i changes over time t as follows:

$$\begin{aligned}\left(\frac{ds_i}{dt}\right)_1 &= p\left\{(m-n)\frac{s_i}{\sum_w s_w}(1+\delta) + \sum_{l\in\Gamma(i)} \frac{s_l}{\sum\limits_w s_w} n \frac{w_{il}}{s_l}(1+\delta)\right.\\ &\quad + \sum_{j\in\Gamma(i)} (m-n)\frac{s_j}{\sum_w s_w}\delta\frac{w_{ij}}{s_j}\\ &\quad \left. + \sum_{j\in\Gamma(i)} \left[\left[\sum_{l\in\Gamma(j)} \frac{s_l}{\sum_w s_w} n \frac{w_{jl}}{s_l}\right]\delta\frac{w_{ij}}{s_j}\right]\right\}\\ &= pm\frac{s_i}{\sum\limits_w s_w}(1+2\delta). \end{aligned} \tag{6}$$

(ii) When the new node obtains local network information, s_i changes over time t as follows:

$$\begin{aligned}\left(\frac{ds_i}{dt}\right)_2 &= (1-p)\frac{M}{m_0+t}\left\{(m-n)\frac{s_i}{\sum_{w\in\mathrm{Local}} s_w}(1+\delta)\right.\\ &\quad + \sum_{l\in\Gamma(i)} \frac{s_l}{\sum_{w\in\mathrm{Local}} s_w} n \frac{w_{il}}{s_l}(1+\delta)\\ &\quad + \sum_{j\in\Gamma(i)} (m-n)\frac{s_j}{\sum_{w\in\mathrm{Local}} s_w}\delta\frac{w_{ij}}{s_j}\\ &\quad \left. + \sum_{j\in\Gamma(i)} \left[\left[\sum_{l\in\Gamma(j)} \frac{s_l}{\sum_{w\in\mathrm{Local}} s_w} n \frac{w_{jl}}{s_l}\right]\delta\frac{w_{ij}}{s_j}\right]\right\}\\ &= (1-p)\frac{M}{m_0+t} m \frac{s_i}{\sum_{w\in\mathrm{Local}} s_w}(1+2\delta). \end{aligned} \tag{7}$$

The variation of s_i over time t can be obtained:

$$\frac{ds_i}{dt} = pm\frac{s_i}{\sum_w s_w}(1+2\delta) + (1-p)m\frac{M}{m_0+t}\frac{s_i}{\sum_{w\in\mathrm{Local}} s_w}(1+2\delta). \tag{8}$$

When the node obtains local network information, at time t, $m \leq M \leq m_0 + t$. Next, we will discuss three cases of local-world selection: $M = m_0 + t$, $M = m$ and $m < M < m_0 + t$:

Case A. $M = m_0 + t$

Under this case, the new node obtains global network information. The model is the clustering-coefficient-adjustable BBV model.[12]

Case B. $M = m$

We assume that $\sum_{w\in\text{Local}} s_w = \langle s_k\rangle M \approx s_i m$, where $\langle s_k\rangle$ represents the average weight of the whole network. Then:

$$\begin{aligned}\frac{ds_i}{dt} &= p\frac{1+2\delta}{2+2\delta}\frac{s_i(t)}{t} + (1-p)\frac{(1+2\delta)m}{m_0+t}\\ &\approx p\frac{1+2\delta}{2+2\delta}\frac{s_i(t)}{t} + (1-p)\frac{(1+2\delta)m}{t}.\end{aligned} \tag{9}$$

Given $a = p\frac{(1+2\delta)}{2+2\delta}$ and $b = (1-p)(1+2\delta)m$, initially, $s_i(t_0) = m$, then:

$$s_i(t) = -\frac{b}{a} + \left(m + \frac{b}{a}\right)\left(\frac{t}{t_i}\right)^a. \tag{10}$$

Thus, the probability that $s_i(t)$ is less than s is as follows:

$$p(s_i(t) < s) = p\left(t_i > t\left(\frac{m+b/a}{s+b/a}\right)^{1/a}\right). \tag{11}$$

Besides, time t follows the uniform distribution:

$$p(t_i) = \frac{1}{m_0+t}. \tag{12}$$

So, the node strength probability distribution is as follows:

$$\begin{aligned}p(s_i(t) < s) &= 1 - p\left(t_i \leq t\left(\frac{m+b/a}{s+b/a}\right)^{1/a}\right)\\ &= 1 - \frac{t}{m_0+t}\left(\frac{m+b/a}{s+b/a}\right)^{1/a}.\end{aligned} \tag{13}$$

The probability that the node strength s is as follows:

$$p(s) = \frac{\partial p(s_i < s)}{\partial s} = \frac{1}{a}\frac{t}{m_0+t}\frac{(m+b/a)^{1/a}}{(s+b/a)^{1+1/a}}. \tag{14}$$

When $t \to \infty$, node strength distribution is approximately $p(s) = s^{-\lambda}$, exponent $\lambda = 1 + \frac{2\delta+2}{p(1+2\delta)}$.

Case C. $m < M < m_0 + t$

Under this case, on one hand, if the new node obtains global network information, the average weight approximates to $\langle s_k\rangle = 2m(1+\delta)t/(m_0+t)$. On the other hand, if the new node obtains local network information, without loss of generality, we assume that the average weight is m. Taking $\sum_{w\in\mathrm{Local}} s_w = [p\langle s_k\rangle + (1-p)m]M$ into Eq. (7), we can get:

$$\frac{ds_i}{dt} = \left[p\frac{1+2\delta}{2+2\delta} + (1-p)\frac{1+2\delta}{p(2+2\delta)+(1-p)}\right]\frac{s_i(t)}{t}. \tag{15}$$

Similarly, $p(s) = s^{-\lambda}$, $\lambda = 1 + \frac{1}{\beta}$, where $\beta = p\frac{1+2\delta}{2+2\delta} + (1-p)\frac{1+2\delta}{p(2+2\delta)+(1-p)}$. When $p = 1$, $\lambda = \frac{3+4\delta}{1+2\delta}$, the new node obtains global network information, under this condition, the model is the clustering-coefficient-adjustable BBV model.[12] When $\delta = 0$, $\lambda = \frac{p^2+p+4}{p^2-p+2}$, edge weights will stop evolving at this moment, then the model turns into an unweighted scale-free network.[13] Besides, every time an edge is added, the total weights of the system will increase by $2+2\delta$, so $\sum_w s_w \approx 2m(1+\delta)t$. Under the initial configuration $k_i(t=i) = s_i(t=i) = m$ and $\frac{dk_i}{dt} = m\frac{s_i}{\sum_w s_w}$, one can find that there is a linear relationship between node strength and degree, thus the node degree distribution follows power-law distribution $p(k) \propto k^{-r}$. Similarly, if a new node is connected to the node i or node j, w_{ij} will change, then: $\frac{dw_{ij}}{dt} = pm\frac{\delta}{1+\delta}\frac{w_{ij}}{t} + (1-p)m\frac{M}{m_0+t}\frac{\delta}{1+\delta}\frac{w_{ij}}{t}$. The link (i,j) is created at $t_{ij} = \max(i,j)$ with initial condition $w_{ij}(t_{ij}) = 1$. So, one can easily see that the edge weight distribution follows power-law distribution $p(w) \propto w^{-r}$.

4. Simulation and Analysis

Under initial conditions and parameters shown as follows: $m_0 = 3$, $m = 3$, $m_0 + t = 3000$, $M = 30$ and $\delta = 1$, we analyze the distribution of degree, strength and edge weight, meanwhile, the relationship both between degree and strength and between degree and clustering coefficient are also discussed. Figures 1 and 2 give the degree distributions, strength distributions are shown in Figs. 3 and 4, edge weight distributions are presented in Figs. 5 and 6. From Figs. 1–6, we can see all of them follow power-law distribution, which are consistent with the theoretical results.

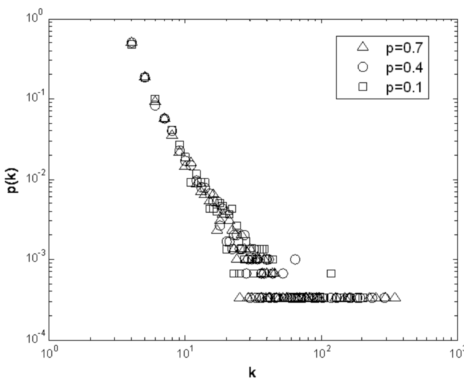

Fig. 1. When $q = 0.5$, the degree distributions with different p values.

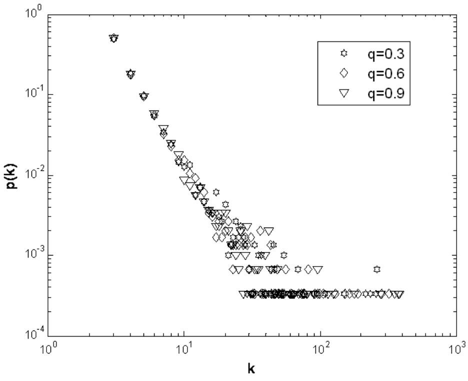

Fig. 2. When $p = 0.5$, the degree distributions with different q values.

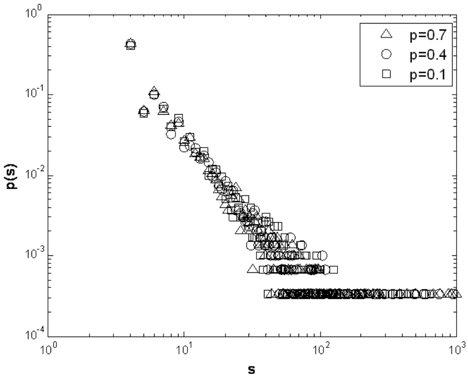

Fig. 3. When $q = 0.5$, the strength distributions with different p values.

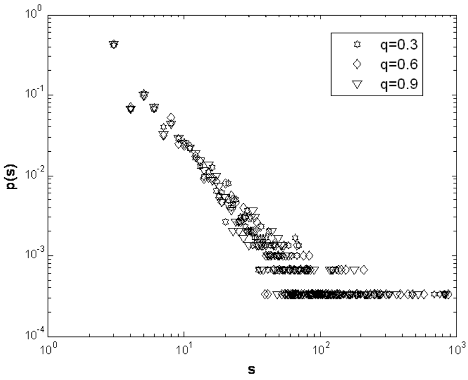

Fig. 4. When $p = 0.5$, the strength distributions with different q values.

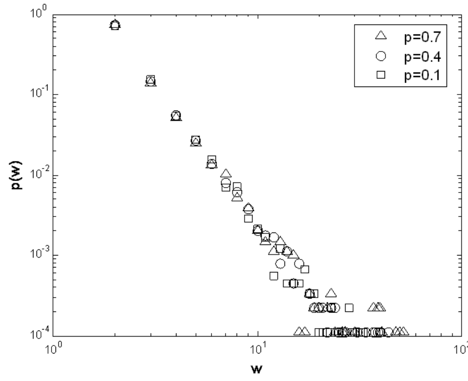

Fig. 5. When $q = 0.5$, the edge weight distributions with different p values.

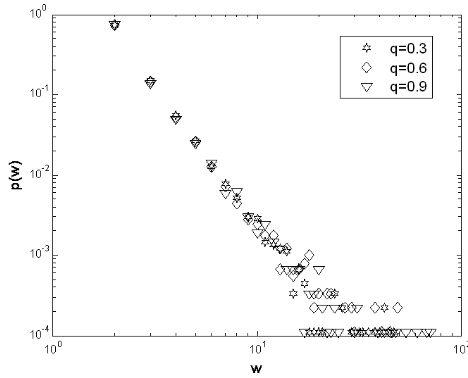

Fig. 6. When $p = 0.5$, the edge weight distributions with different q values.

Besides, Figs. 7 and 8 show the relationship between degree and strength of the model, where a linear relationship between the degree and strength can be found. Here, by adjusting the value of q and p, the strength of the node can increase faster than the node degree, which conforms to the real rail network. In the rail network, the edge weight represents the carrying capacity of a station, the case that nodes with greater degree have larger edge weight than the nodes with small degree is an equation of state that large-scale train stations equip greater carrying capacity than ordinary ones. So, in order to better study the rail network, we can establish a model closer to it by adjusting the values of q and p. Figures 9 and 10 provide the relationship between degree and clustering coefficient of the model, where $c(k)$ is the average clustering coefficient of the node with degree of k, given by the formula (16).

$$c(k) = \frac{1}{Np(k)} \sum_{i/k_i=k} c_i. \tag{16}$$

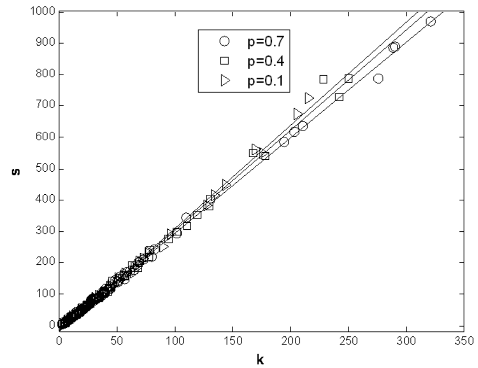

Fig. 7. When $q = 0.5$, the relationship between degree and strength with different p values.

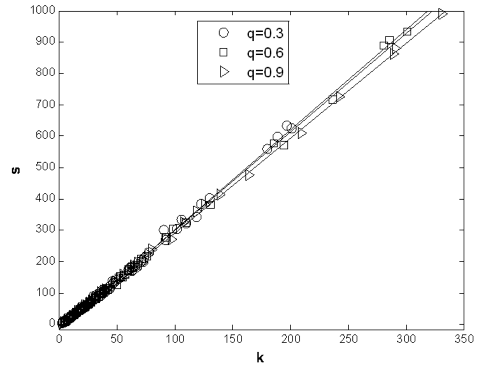

Fig. 8. When $p = 0.5$, the relationship between degree and strength with different q values.

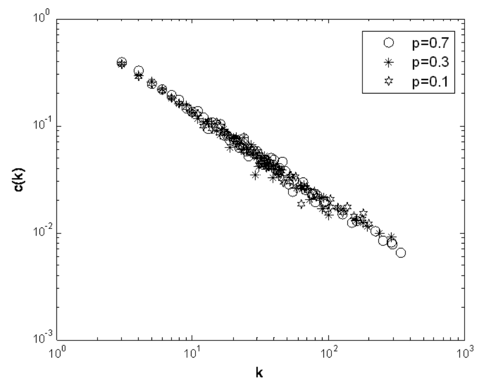

Fig. 9. When $q = 0.5$, $k - c(k)$ relationship with different values of p.

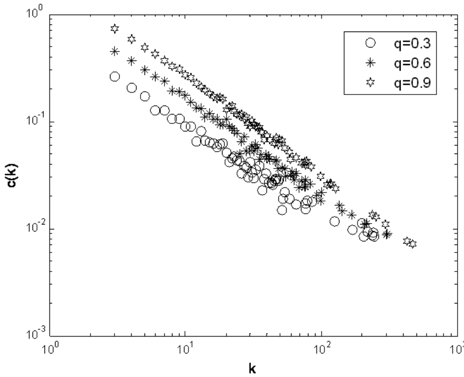

Fig. 10. When $p = 0.5$, $k - c(k)$ relationship with different values of q.

In the BBV model one can adjust the network clustering coefficient by altering extra traffic δ. Then, the traffic across the network will get heavier gradually with the increase of δ. Due to that most real networks have its own maximum traffic tolerance, so it is obviously unreasonable to adjust clustering coefficient by turning extra traffic δ. In our model, the clustering coefficient can be changed by adjusting the local-world selection probability p and triangle coupling probability q. Although significant effect cannot be reached by changing p, while the network clustering coefficient can be increased by improving the triangle coupling probability q. Therefore, our model is more efficient than the original BBV model in adjusting clustering coefficient.

5. Analysis of Virus Spread Behavior on Our IBBV Model

Under initial conditions and parameters shown as follows: $m_0 = 3$, $m = 3$, $m_0 + t = 3000$, $\delta = 1$, $p = 0.5$ and $q = 0.5$, when $M = 5$, SI model is used to study virus spread behavior on our IBBV model. Here, the infection probability is defined as $\lambda_{ij} = \frac{w_{ij}}{s_i}$, $0 < \lambda_{ij} < 1$. So, different edge weights between nodes will affect the virus spread. The virus spread rate is defined as the change rate of infected individuals in the network:

$$v_{\inf}(t) = \frac{di(t)}{d(t)} \approx \frac{I(t) - I(t-1)}{N}. \tag{17}$$

Initially, 1% nodes in our network are infected randomly, the virus spread behavior over time is observed. The variation of virus infection density during the steady state is shown in Fig. 11 as the solid line, and the variation of virus spread rate is shown in Fig. 12 as the asterisk line. Under the same initial conditions, the infection density and the virus spread rate are obtained with the different values of local-world size M which is set to be 5, 20, 50 and 100, respectively shown in Figs. 11 and 12. It can be seen that the infection density and the virus spread rate will both increase with the

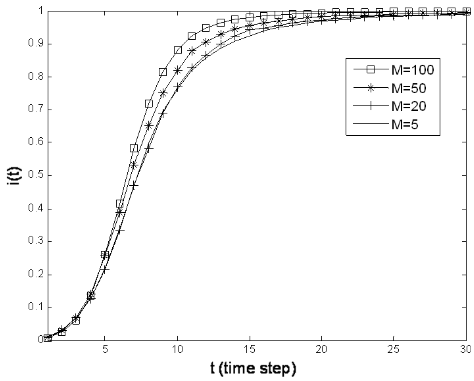

Fig. 11. Variation of infection density during the steady state.

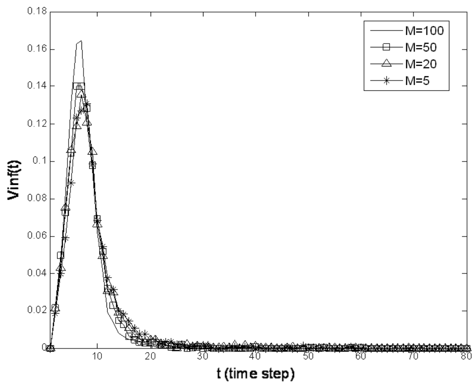

Fig. 12. Variation of virus spread rate.

increasing of local-world size M. From above results, we can find that reducing the size of local-world has a great inhibited effect on virus spread both from quantity and speed. Therefore, controlling the scope of individual exposure plays an active role in inhibiting the virus spread in a virus outbreak.

6. Conclusions

In this paper, we have constructed a model of growing weighted networks that not only considers the effect of the coupling between topology and weights dynamics, also include local-world selection and triad formation step. We investigated in detail the local-world selection mechanisms and carried out both theoretical analysis and numerical simulation. The model produces graphs which display nontrivial complex and scale-free behavior, particularly, different quantities such as strength, degree, and weights are distributed according to power laws with exponents which are not universal and depend on the specific parameters that can adjust the local microscopic weights dynamics. The model also presents linear relationships both between the degree and strength and between the degree and the clustering coefficient. Particularly, the model is equipped with an ability to accelerate the speed increase of strength exceeding that of degree, which can describe the property of the rail network that large-scale train stations equip greater carrying capacity than ordinary ones. Besides the model is more sound and efficient in tuning clustering coefficient than the original BBV model. In addition, virus spread process on our IBBV model has been investigated. We find that the infection density and the virus spread rate will increase with the increasing of local-world size M, which may be useful to comprehensively understand the virus spread behavior on weighted networks.

Acknowledgments

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