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Three-layered supernetwork evolution model and the application for China-world's top 500 enterprises supernetwork

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Network of network (NON) or so-called supernetwork extensively exists in the real world. However, so far the definition of NON is not mutually recognized, relevant theory is rather lacking. In order to reveal certain characteristics of NON, we proposed four kinds of three-layered supernetwork evolution models (TLSEM) based on WS small-world and BA scale-free model, and defined two kinds of layer cross-degrees as new measures of cooperative-competition relationship for different layer nodes. The idea and methods of TLSEM are applied to the construction and analysis of China-world's top 500 enterprises supernetworks as a typical empirical example. The analytical results show that the layer cross-degree is better description than other network characteristics, and TLSEM may lay a certain foundation and extend to study more multilevel supernetworks.

Keywords: NON; top 500 enterprises; layer cross-degree.

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1. Introduction

People live in a world filling with a variety of networks, such as Internet, WWW, brain networks and so on.[1–4] Network of network (NON) or supernetwork widely exists in the real world, it is a true and perplexing representation of the real world network. The main features of NON may be as follows: in a status of "you have me, and I am in you", networks nest networks, network nodes are maybe networks in itself, as well as multilevel, multidimensional, multiproperty, multiobjective characteristic and so on.[5–9] But NON has not general acceptance definition and relevant theory is rather lacking.[10–14] So far the theoretical research of NON has many

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challenges, understanding above multilevel network properties is very necessary, revealing the dynamical evolution of real systems which are made up of two or more interconnected networks has practical implications. Supernetwork is an integration of various types of self-organization networks, such as high-tech networks, and military networks are typical examples of supernetwork. We cannot simply use previous network approach to analyze supernetwork, although real networks have been shown to have scale-free and small-world phenomenon.[1–4] Supply chain supernetwork equilibrium model is a typical example of supernetwork, the sub-graph centrality degree and clustering coefficient characteristics were studied.[15–17] These efforts for exploring supernetwork have some reference value, but the model construction methods on supernetwork are most based on some special objects, the whole subject is still in the exploratory stage.

Therefore, in order to understand and reveal certain characteristics of NON which is made up of two or more interconnected networks, we need a new idea of description. This paper proposed and constructed four kinds of three-layered supernetwork evolution models (TLSEM) based on WS small-world model and BA scale-free model. For TLSEM, we not only calculated the general topological properties, such as average clustering coefficient, average shortest path length and degree-degree correlation coefficient, but also defined and studied two kinds of layer cross-degrees as new characteristic quantities, which are used to analyze the cooperation-competition relationships among different layer nodes of NON. Then, we extended the theoretical research of TLSEM to calculate and analyze China-world's top 500 enterprises supernetwork with three layers. The numerical simulation results show that for the supernetwork models and the empirical analysis the layer cross-degree is a better description than other characteristics, and can be used to analyze the mutual cooperation-competition layer relationships among nodes of different layers. These results may lay a certain foundation for theoretical analysis and application research for more multilevel supernetworks.

2. The Analysis and Construction of Supernetwork Evolution Model

Real networks often have small-world and scale-free characteristics. Based on Barabási–Albert (BA) and Watts–Strogatz (WS) models, we proposed the following construction method for TLSEM: (1) three layers are BA model (TBA); three layers are SW model (TSW); the first and third layer are BA model, the middle layer is SW model (BA-SW); the first and third layer are SW model, the middle layer is BA model (SW-BA). The number of nodes per layer is 1000. (2) There are some connection edges between the first and second layer or the second and third layer, the connection probability is LP. The nodes of the first layer do not connect with the nodes of the third layer. (3) In order to explore the affect of network connections between the upper and lower edges from a different angle, we considered two types of connection patterns: random and preferential.

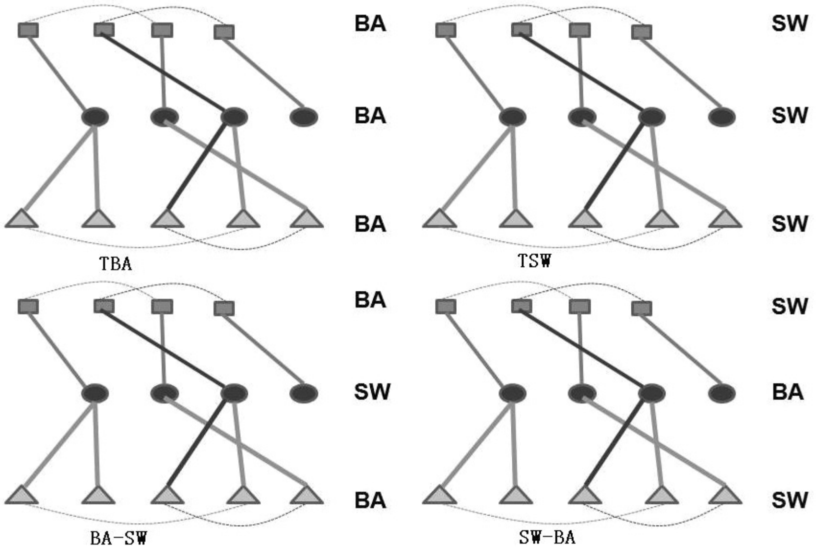


Fig. 1. Three-layered supernetwork models diagram.

For random pattern (is called Rp) each corresponding node is assigned to the random number R. If R is bigger than the connection probability LP, the corresponding nodes connect to each other, otherwise do not connect. For preferential pattern (is called Pp) each node degree is sorted, the node which degree is bigger has greater connection probability.

Four kinds of three layered supernetwork models are constructed by above rules and shown in Fig. 1. First, we calculated some general topological properties, such as average clustering coefficient, average shortest path length and degree–degree correlation coefficient (see in Figs. 2–4). The solid lines are random pattern, the dotted lines are preferential pattern. The equation of the average clustering coefficient c is:

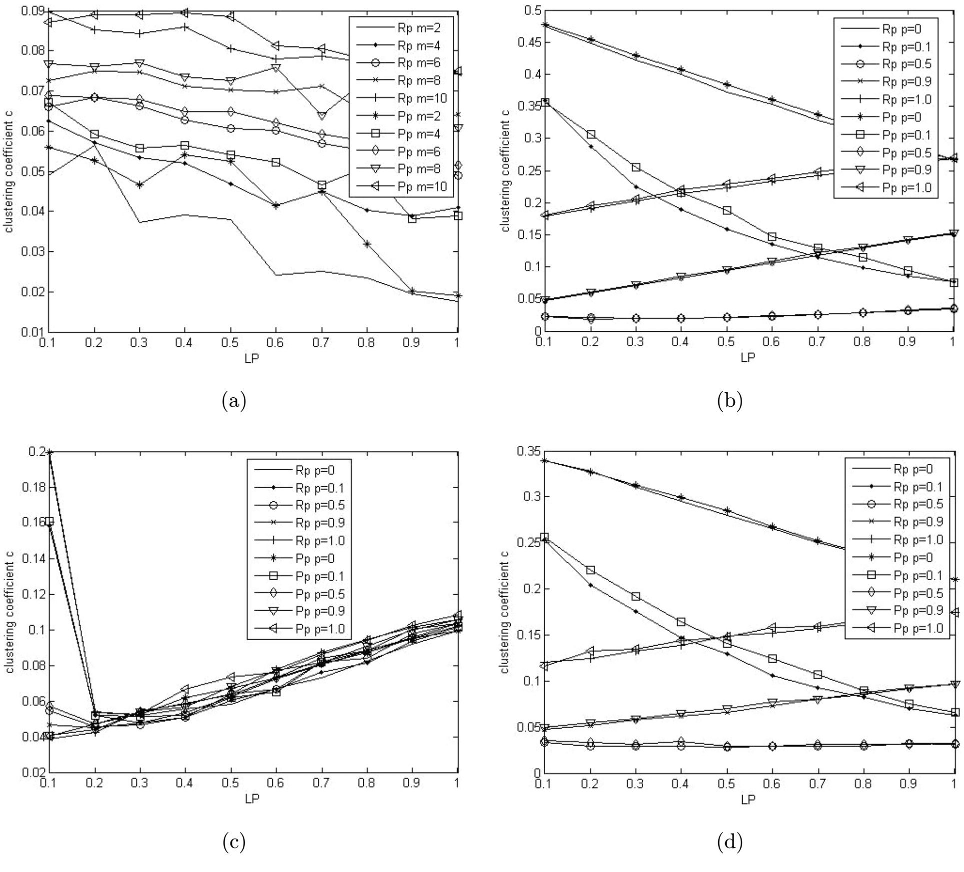
$$c = \frac{1}{N} \sum_{i=1}^{N} \frac{2E_i}{k_i(k_i - 1)}. \tag{1}$$

The calculation results of the average clustering coefficient c are shown in Fig. 2. For TBA model the c decreases as LP increases, the clustering coefficient of preferential pattern is bigger than random pattern. When for TSW model the evolution probability p of small-world is relatively small, the c decreases with LP increasing. When the evolution probability p is quite big, the c increases with LP increasing. The calculation results of Rp and Pp are very similar. It is noted that for BA-SW model the c has V-curve characteristic and a turning point at $LP = 0.2$. When $LP < 0.2$, the c decreases with LP increasing. When $LP > 0.2$, the c increases with LP increasing. The results of SW-BA model are similar to the results of TSW model.

The average shortest path L reflects the connectivity of whole network, it is defined as follow:

$$L = \frac{\sum_{i \neq j} d_{ij}}{N(N-1)}. \tag{2}$$

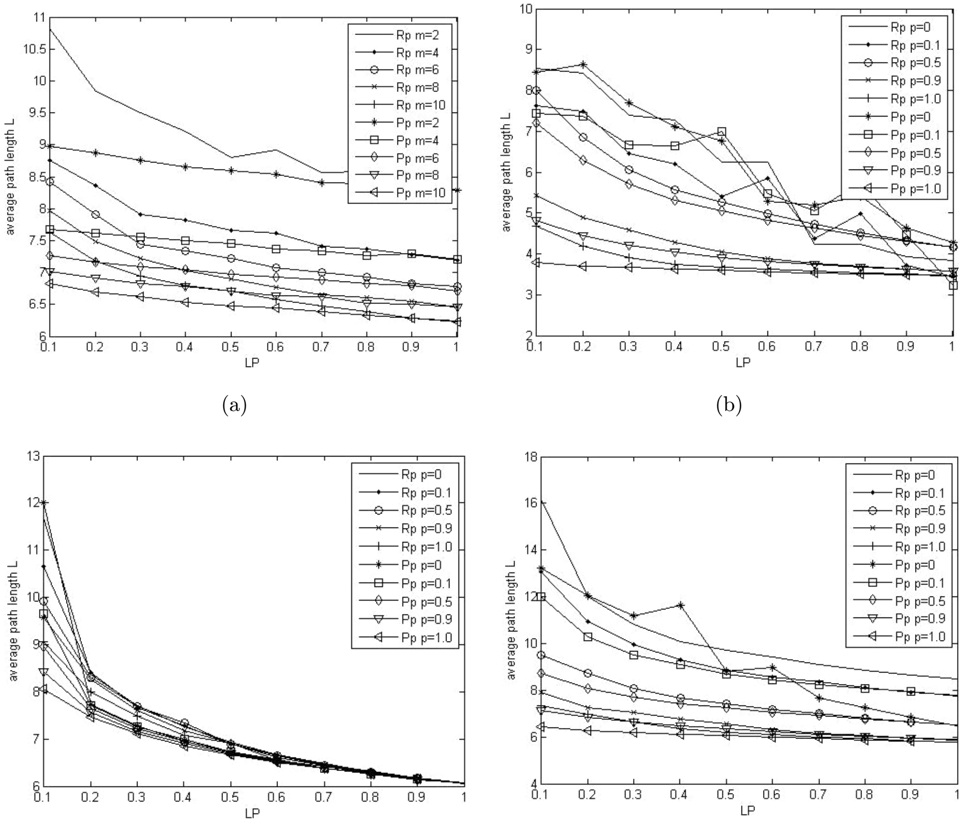
It is known from Fig. 3 that all the average shortest path lengths L reduce as the connection probability LP increases. For TBA model the L of preferential pattern is smaller than random pattern. When m of BA increases, the L gradually reduces. For BA-SW model the L decreases with LP increasing and the difference between preferential pattern and random mode is quite small. The L of TSW and SW-BA model reduce with the evolution probability p increasing.



(a) (b)

(c) (d)

Fig. 2. The average clustering coefficient c of TLSEM. (a) TBA, (b) TSW, (c) BA-SW, (d) SW-BA.



(a)

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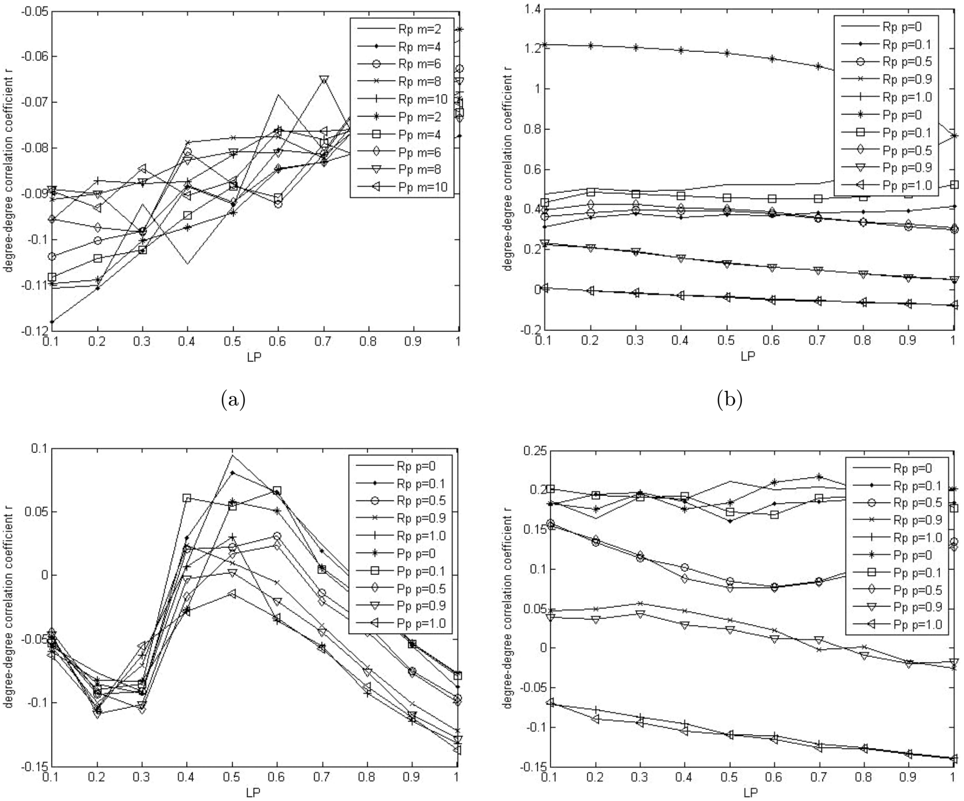
Fig. 3. The average shortest path length L of TLSEM. (a) TBA, (b) TSW, (c) BA-SW, (d) SW-BA.

The degree–degree correlation coefficient r reflects correlation between degrees and is given by:

$$r = \frac{e_t^{-1}\sum_m p_m q_m - [e_t^{-1}\sum_m \frac{1}{2}(p_m + q_m)]^2}{e_t^{-1}\sum_m \frac{1}{2}(p_m^2 + q_m^2) - [e_t^{-1}\sum_m \frac{1}{2}(p_m + q_m)]^2}\,. \tag{3}$$

The calculation results of the degree–degree correlation coefficient r are shown in Fig. 4. Although the r of TBA model has some fluctuations, but the r is always less than zero when LP increases, so TBA model is negative correlation. For TSW model the r decreases with LP increasing and is always greater than zero, so TSW model is positive correlation. The calculation results of BA-SW model are very complex, there are two extreme points which are corresponding to $LP \in [0.2, 0.3]$ and $LP \in [0.5, 0.6]$. The results of SW-BA model are similar to the results of TSW model, only when $p = 1$ the model is negative correlation.



(a)

(b)

(c)

(d)

Fig. 4. The degree–degree correlation coefficient r of TLSEM. (a) TBA, (b) TSW, (c) BA-SW, (d) SW-BA.

3. The Analysis of Layer Cross-degree on Supernetwork Models

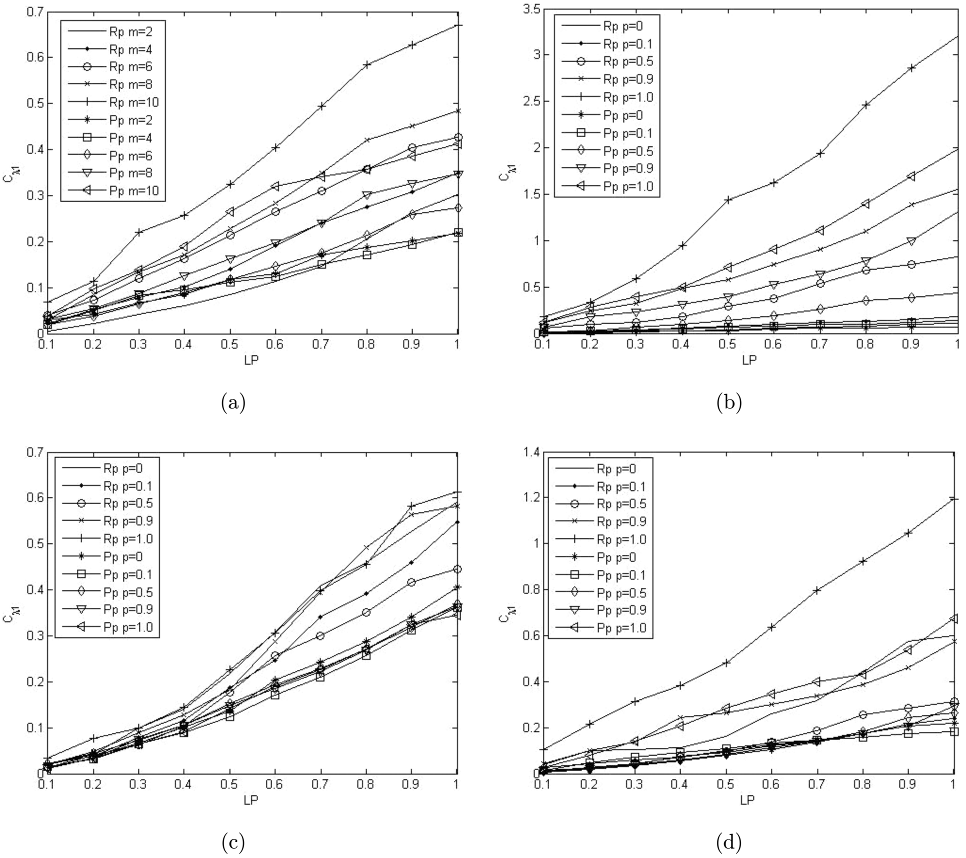
Since the research of conventional complex network characteristics ignores the influences of the edges between different layers, obviously such abstraction for any real topological structure is not enough for comprehensive description, especially for the real supernetwork. In order to improve analysis of supernetwork models, we introduced a new characteristic quantity so-called as “layer cross-degree”. In TLSEM model there are five different types of edges: the edges in the first layer (U), the edges in the second layer (M), the edges in the third layer (L), the edges between the first and second layer (UM), the edges between the second and third layer (ML). When we consider the shortest path length from node i to j, the number of five different types of edges is denoted by U_{ij}, UM_{ij}, M_{ij}, ML_{ij}, L_{ij}, respectively. The term N represents the total number of nodes. Two kinds of layer cross-degrees are defined as follows:

$$C_{\lambda 1} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{UM_{ij} + ML_{ij}}{U_{ij} + L_{ij} + M_{ij} + UM_{ij} + ML_{ij}}, \qquad (4)$$

$$C_{\lambda 2} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{UM_{ij} + ML_{ij}}{U_{ij} + L_{ij} + M_{ij}}. \qquad (5)$$

If $UM_{ij} > 0$ there are cooperation-competitive relationships between the first and second layer, if $ML_{ij} > 0$ there are cooperation-competitive relationships between the second and third layer. If $C_{\lambda 1} = C_{\lambda 2} = 0$, this may be called the layer cross critical point or the emergence point of cooperation-competition. When $C_{\lambda 1} > 0$ or $C_{\lambda 2} > 0$, cooperation-competition relationship may emerge.

The calculation results of the layer cross-degrees are shown in Figs. 5 and 6, all the layer cross-degrees of multilevel supernetwork models linearly grow with the



(a) (b)

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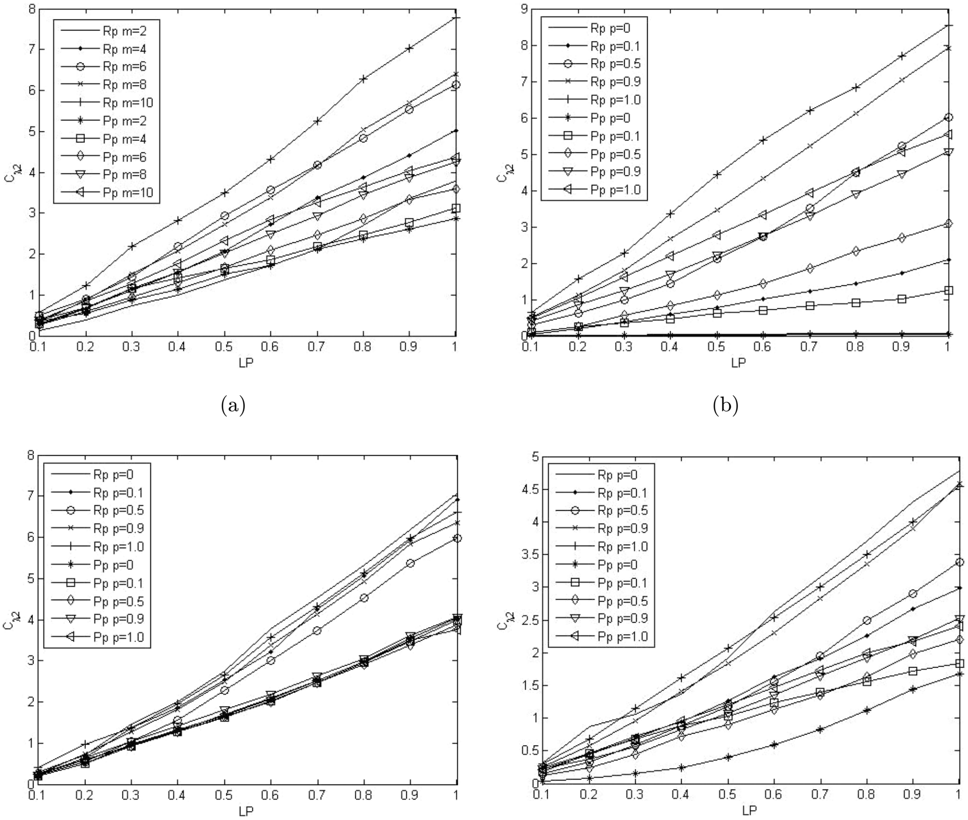
Fig. 5. The layer cross-degree $C_{\lambda 1}$ changes with LP. (a) TBA, (b) TSW, (c) BA-SW, (d) SW-BA.



(a) (b)

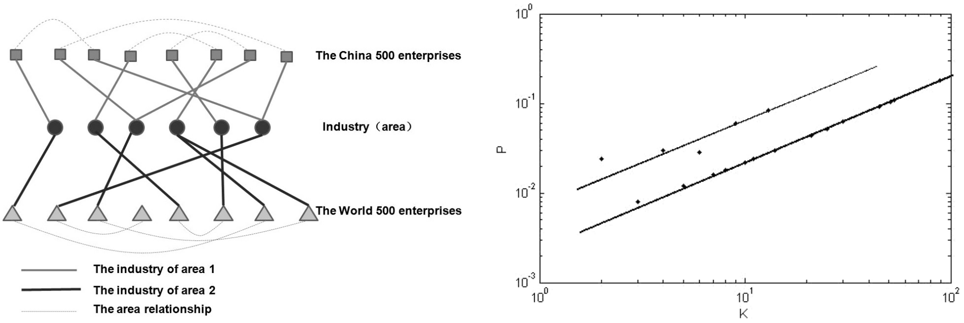
(c) (d)

Fig. 6. The layer cross-degree $C_{\lambda 2}$ changes with LP. (a) TBA, (b) TSW, (c) BA-SW, (d) SW-BA.

connection probability LP increasing. The biggest slope of $C_{\lambda 1}$ is 3.2, and the smallest slope is 0.05. The biggest slope of $C_{\lambda 2}$ is 8.5, and the smallest slope is 0. The curves of the layer cross-degree $C_{\lambda 1}$ and $C_{\lambda 2}$ are relatively big in random pattern, the curve slopes of TSW are the biggest when p is maximum. The results show that when the edges of each layer are random connection, the growth of the layer cross-degree is the fastest with LP increasing, and the cooperation-competition relationships between nodes of different layers emerge faster.

4. Application Examples for China-world's Top 500 Enterprises Supernetwork

In previous works,[18,19] we have studied China's and world's top 500 enterprises network models and analyzed the characteristics of these networks. In order to combine with previous works, and systematically study cooperative-cooperation relations between China's and the world's top 500 enterprises, we applied above



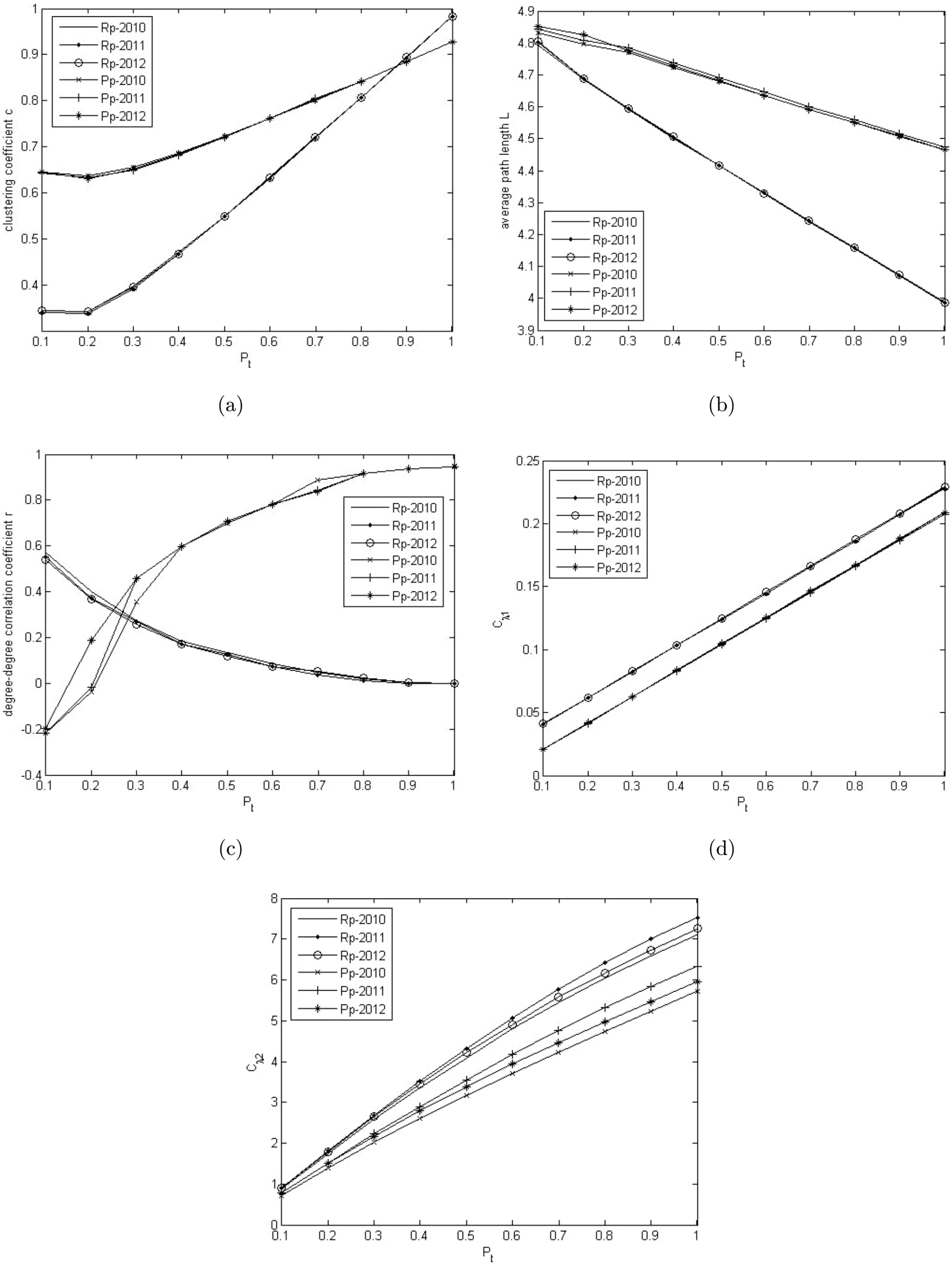
(a) (b)

Fig. 7. (a) China-world's top 500 enterprises regular supernetwork model, (b) the degree distribution in 2012.

theory model and method to analyze a typical empirical supernetwork so-called China-world's top 500 enterprises supernetwork. We collected relevant data from 2010 to 2012. We assumed that the first layer nodes are China's top 500 enterprises, the second layer nodes are the industries or areas involved, and the third layer nodes are the world's top 500 enterprises. The enterprises in the first and third layer first connect each other if they belong to the same area or industry. If the enterprises in the first and second layer or the second and third layer belong to the same industry, they connect each other. According to above rules, we constructed China-world's top 500 enterprises regular supernetwork. The schematic diagram of the regular supernetwork is shown in Fig. 7(a). The degree distribution of the regular supernetwork is the same as high-tech networks which we have studied, they are both multiple power-law distribution (see in Fig. 7(b)).

Besides the regular connections among China-world's top 500 enterprises, based on above research results, we introduced random and preferential attachments between the first and second layer or the second and third layer. We defined a threshold probability P_t for each node, if the first layer nodes connect to the third layer nodes depends on the threshold value P_t. The normalization calculation results of China-world's top 500 enterprises operating receipt are the preferential connection probability. If the connection probability is bigger than the threshold value P_t, China-world's top 500 enterprises connect each other, otherwise do not. We calculated the complex network characteristics and the layer cross-degrees of China-world's top 500 enterprises random and preferential supernetwork model, which are shown in Fig. 8.

The average clustering coefficient c increases with the threshold value P_t increasing in 2010–2012. When $P_t \leq 0.9$, the clustering coefficient c of preferential pattern is bigger than random pattern. When $P_t > 0.9$, the clustering coefficient c of preferential pattern is smaller than random pattern. The average shortest path length L decreases with P_t increasing, the average shortest path length L of preferential pattern is smaller than random pattern. The degree-degree correlation



(a) (b)

(c) (d)

(e)

Fig. 8. The calculation results of China-world's top 500 enterprises supernetwork models from 2010 to 2012. (a) the clustering coefficient c, (b) the average shortest path length L, (c) the degree-degree correlation coefficient r, (d) $C_{\lambda 1}$, (e) $C_{\lambda 2}$.

coefficient r of preferential pattern increases with P_t increasing, and the supernetwork is positive correlation. The degree-degree correlation coefficient r of random pattern decreases with P_t increasing. When $P_t \leq 0.3$, the degree-degree correlation of preferential pattern is weaker than random pattern. When $P_t > 0.3$, the degree-degree correlation of preferential pattern is stronger than random pattern.

The layer cross-degree $C_{\lambda 1}$ of China-world's top 500 enterprises supernetwork linearly increases and $C_{\lambda 2}$ nonlinearly increases when P_t increases. Two kinds of layer cross-degrees of preferential pattern are both bigger than random pattern. These results suggest that average clustering coefficient, average shortest path length and degree–degree correlation coefficient for supernetworks are irregular, and they are not suitable to be used to measure the relationship between China-world's top 500 enterprises. However, the layer cross-degrees are able to clearly reflect cooperation-competition relationships between China-world's top 500 enterprises since they appear regularly changing for preferential pattern and random pattern. These calculation results are in keeping with the theory model research.

5. Conclusion

The NON or supernetwork research is only just beginning to be explored, currently it still stays in tentative exploration stage. In this paper, we used the most classic small-world and scale-free model to construct four kinds of TLSEM, introduced random attachment and preferential attachment mean, and analyzed the general topology characteristics of TLSEM. In order to more correctly describe the relationships between nodes of multilevel supernetwork models, we proposed two kinds of layer cross-degree definitions of TLSEM, which are used as a new characteristic quantity for better research purposes. To prove theoretical model, we extended above idea and method to study China-world's top 500 enterprises random and preferential supernetworks. The research results show that for supernetworks the layer cross-degrees are better described than conventional characteristics for analyzing cooperation-competition relations between different layer nodes. Therefore, the idea and method of TLSEM may preferably describe a kind of supernetworks at least, lay a foundation for the multiplex networks and be of potential application for NON.

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