International Journal of Modern Physics C
Vol. 25, No. 5 (2014) 1440002 (10 pages)

DOI: 10.1142/S0129183114400026

Bid distribution derived from consistent mixed strategy in lowest unique bid auction

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Received 30 June 2013
Accepted 30 September 2013
Published 16 December 2013

The Lowest Unique Bid Auction (LUBA) booms recently through the Internet. A typical distribution pattern of bid price in this reverse auction has been found and needs to be interpreted. The distribution curve is a decreasing one whose slope has a close relationship with the number of agents participating in the auction. To explain this stylized fact, we develop a model assuming that agents prefer to bid on the price at which the probability of winning is higher. The bid distributions of actual auctions with the number of agents less than 200 can be fitted very well using the parameters for the value of items and the number of bids. When this number becomes larger, however, a deviation occurs between prediction and empirical data, which can be adjusted by introducing cognitive illusion of the bid number.

Keywords: Lowest unique bid auction; bid distribution; mixed strategy; cognitive illusion.

PACS Nos.: 02.50.Le, 89.65.-s.

1. Introduction

The last decade has witnessed an emerging and expansive research attempt in understanding human behavior.[1–5] Auction, a typical socioeconomic activity, has attracted much attention.[6] In history, English Auction, Dutch Auction, Sealed-Highest Price Auction, Sealed-Second Highest Price Auction and many others came out in sequence contributing to the boost of this industry. Then theories and models were proposed to understand the human decision-making mechanism when a

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person's bid concerns with his or her own constrains as well as complex situations derived from others' behavior.[7–11] Recently, a new kind of auction, the Lowest Unique Bid Auction (LUBA), has spread all over the world, especially through the Internet. In this auction, the winner makes the lowest bid among unique ones instead of the highest one as usual. It attracts so many people to get an item of high value at a relatively low cost. Definitely, this new game also brings us a challenging research topic.

Actually, primary theoretical study was focused on the Lowest Unique Positive Integer (LUPI) game, whose rules and distributions are similar to LUBA. Zeng *et al.* analyzed LUPI by game theory.[12] According to their research, in both random and rational selections, the winning strategy was always selecting the smallest one. Then, other researchers studied this issue in terms of Nash Equilibrium.[13,14] Besides pure theoretical analysis, Östling *et al.* studied LUPI with data from Limbo, a four-week LUPI game started on 29th of January 2007 in Sweden.[15] They deduced theoretical equilibrium and found the empirical results deviated from equilibrium. They employed a cognitive hierarchy (CH) model to explain the deviation.

Thanks to the development and popularity of the Internet, LUBA grows fast, which provides ample data of such kind of auctions benefiting this research a lot. Eichberger and Vinogradov used some of them to compare actual behavior of players with theoretical prediction.[16] The easy access of data also spurred more theoretical attempts. Radicchi *et al.* found the Levy flight search strategies agents adopted by chronologically sorting bids.[17] Pigolotti *et al.* assumed Poisson-distributed uncertainty about the number of players and proposed a Nash model based on expected payoff independent of bid prices.[18] Many complex and multi-parameter models and algorithms were also developed. Rapoport *et al.* constructed a symmetric mixed-strategy equilibrium (SMSE) and used nonstationary Markov chains to numerically compute the SMSE solution to any desired degree of accuracy.[19] Then, they proposed a new algorithm for computing SMSE solutions to the LUBA.[20] Houba *et al.* thought that the symmetric Nash equilibrium with the lowest expected gains was the maximum in symmetric strategies.[21]

Different from these studies, we develop a model without prior Poisson-distributed assumption in the number of agents participating in these auctions. Moreover, the model is simple but more reasonable based on the value of items and the number of bids. Last but not the least, we test the model with empirical data downloaded from the Internet. The model reproduces the probability distribution of LUBA bid price very well, especially when the number of agents is not too large (ordinarily less than 200). When the number becomes bigger, only slight adjustment is needed.

The structure of the paper is as follows. In the next section, we introduce rules, data source and features of the LUBA. In Sec. 3, the model is proposed and its characteristics are predicted. In Sec. 4, we introduce the cognitive error, where agents seem to perform in small numbers instead of actually large ones. Our conclusion is drawn in Sec. 5.

2. Rules and Data of LUBA

2.1. *Rules*

The winner in the LUBA makes the lowest bid among unique ones, which is different from other auctions. This peculiarity can be characterized by the following basic rules:

(i) The value of the item being auctioned v is known to all agents during the whole auction process.
(ii) In each auction, the total number of bid N is fixed, so that the auction will end once N is reached.
(iii) Bids (k, $\underline{b} \leq k \leq \bar{b}$) are integers. Generally, $\underline{b}$ equals to 1 and $\bar{b}$ equals to v.
(iv) Every bid requires an entry fee c. The sum of the entry fees minus the cost gives the revenue of the organizer.

Finally, the winner is the agent who bids the lowest among unique ones.

For the purpose of illustration, we give a hypothetical example shown in Fig. 1 to help understand these rules. There are six possible bids ranging from \$1 to \$6, but nobody bids at \$2 and only \$3 and \$5 are unique. Between these two prices, \$3 is the winning bid because it is the lowest one.

2.2. *Data source*

The data is collected from www.auctionair.co.uk. The information has two parts. One is the major information of the auction, including its ID and value. The other is the detailed bid messages, including the price and time of each bid.

Four cases we choose vary from mobile phones to digital cameras with different value, even including a SUV (see Table 1). Let us take item A, a 16 GB Apple iPhone 4, for an example. Its value is £499. We choose 19 samples in which lots are always A with a similar number of bids. The average number is 135 recorded in the column of Bid Number (N).

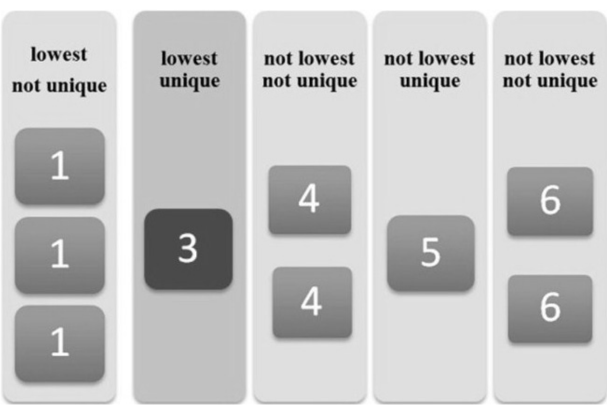


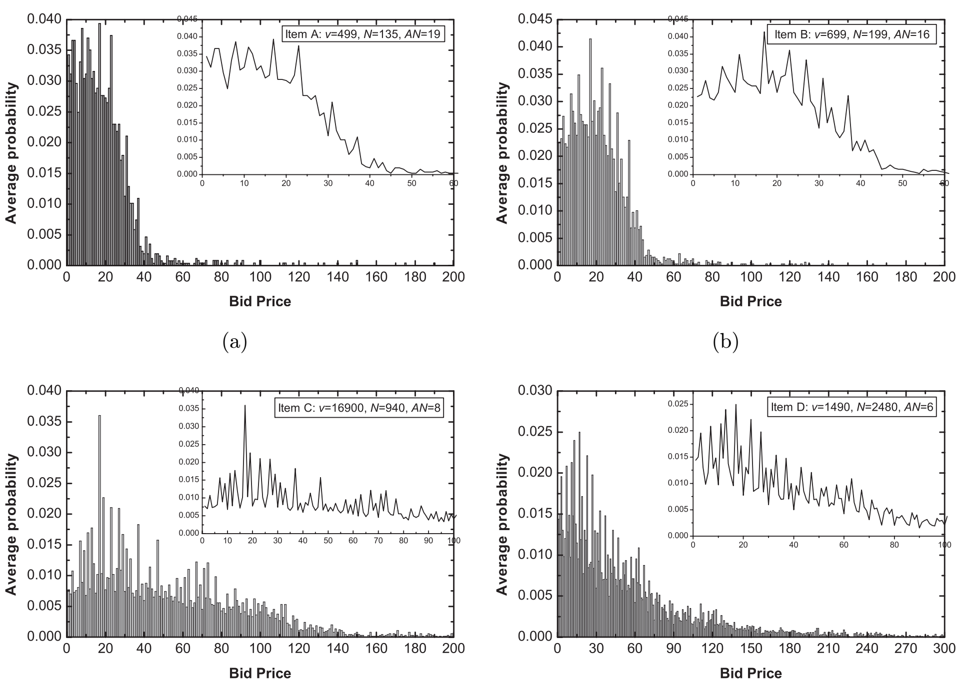
Fig. 1. A hypothetical example of LUBA, with the winning bid of \$3.

Table 1. Value of items and bid number in sample LUBA.

Item	Value (v)	Auction number (AN)	Bid number (N)
A	499	19	135
B	699	16	199
C	16 900	8	940 ± 5
D	1 490	6	$2\,480 \pm 19$

2.3. *Bid probability distribution of LUBA*

The bid probability distributions of four items in Table 1 are displayed in Fig. 2. More precisely, we amplify the probability of low bids in each panel. Some typical features of this distribution can be found. Generally speaking, the probability distribution is decreasing as the bid price increases. It can be divided into three regions, the plateau region with a high probability at low prices, the decreasing region at the middle level of prices and the low probability region at high prices. The plateau region mostly lasts from price 1 to 40. When the number of agents increases, the height of this plateau will drop and the decreasing speed in the second region will rise. Further, more expensive the item is, larger the zero probability point is.



(a) (b) (c) (d)

Fig. 2. Bid distributions of four items in Table 1.

3. Model and Fitting

3.1. *Basic model*

We assume that there are N agents in this game. The value of the item v is known to all agents. Each agent bids only once during the process. Without loss of generality, the bid price is limited in $1 \leq k \leq v$. Also, we assume that all the agents are homogeneous in that. They will bid with the same strategy $p(k)$. So when the auction closes, the distribution $p(k)$ of the whole system will be obtained.

We examine the winning probability of a given bid. Let us denote it as k, which means that only one agent bids here. At the same time, nobody or more than one person bid on prices less than k. The probability of no unique bid at the price of 1 to $(k-1)$ is

$$w(k) = \prod_{j=1}^{k-1} \{1 - C_{N-1}^{1} p(j)[1-p(j)]^{N-2}\}. \tag{1}$$

With nobody other than the winner bidding on price k, the probability of k being a winning number is

$$u(k) = [1-p(k)]^{N-1} \prod_{j=1}^{k-1} \{1 - C_{N-1}^{1} p(j)[1-p(j)]^{N-2}\}. \tag{2}$$

Then we assume that agents adopt symmetric mixed strategies, where the larger $u(k)$ is, the larger $p(k)$ is. It can be written as

$$p(k) \propto u(k). \tag{3}$$

This proportionate relationship is reasonable and has been applied for researches in many areas, such as complex network, physics, biology and many other areas.[22–24]

From to (2) and (3), we can get:

$$p(k) \propto [1-p(k)]^{N-1} \prod_{j=1}^{k-1} \{1 - C_{N-1}^{1} p(j)[1-p(j)]^{N-2}\}. \tag{4}$$

Finally, we get the iteration equation:

$$\frac{p(k+1)}{[1-p(k+1)]^{N-1}} = \frac{p(k)}{[1-p(k)]^{N-1}} \{1 - (N-1)p(k)[1-p(k)]^{N-2}\}. \tag{5}$$

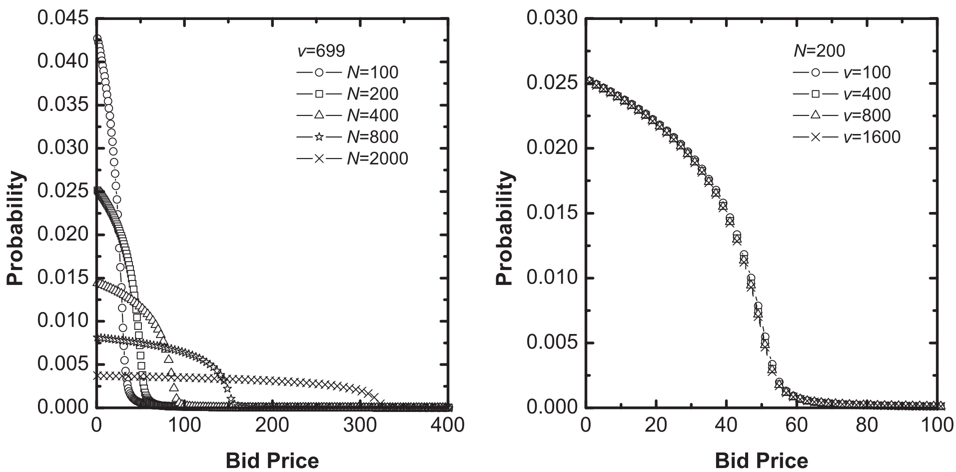
Equation (5) shows that, if probability $p(1)$ is given, all the other $p(k)$ will be deduced. Thanks to the condition $\sum_{k=1}^{v} p(k) = 1$, we are ensured that $p(1)$ is determined and the solution of (5) is unique.

It is easy to demonstrate that $p(k)$ is decreasing as the bid price k increases. Since $\{1-(N-1)p(k)[1-p(k)]^{N-2}\} < 1$, the left side of (5) is always smaller than $\frac{p(k)}{[1-p(k)]^{N-1}}$. So the value of $\frac{p(k)}{[1-p(k)]^{N-1}}$ will fall when k becomes larger, which in turn leads to a decrease of $p(k)$, as $\frac{p(k)}{[1-p(k)]^{N-1}}$ is a monotonously increasing function of $p(k)$. This decreasing tendency has also been pointed out by several other researchers.[12,18,19]

3.2. *Model characters*

There are two parameters in our model. One is N, the total number of agents or bids (each agent can bid only once). The other is v, the value of items or potential maximum bid price (rational agent would not bid at higher price than that). Here, we discuss how these two parameters affect the bid distribution.

Figure 3 shows the predicted bid distribution of the item with the same value but different numbers of agents and the same number of agents but different value. For each curve, we can find that with the increasing bid price, the bid probability gradually decreases from relatively high values and ultimately approaches nearly to zero. In Fig. 3(a) of the same value 699 but different numbers of bids (100, 200, 400, 800 and 2 000), the probability distribution curves display different behavior. As agents' number becomes larger, evidently the initial probability $p(1)$ decreases. Meanwhile, the plateau region gets wider and agents will bid at more various prices. There is one possible explanation that agents will be more afraid of bidding confliction with others when the number of agents is relatively big. So they would like to bid at higher instead of lower prices. On the contrary, if the agents are in a small number, they will more concern about the winning chances at low prices and then bid there. Besides this, in Fig. 3(b), the bid distributions for the same number of agents but different items are almost same. There are only tiny differences at the beginning and the end. This is because the agents' purpose is to obtain a high profit at a low



(a) (b)

Fig. 3. The bid distribution with different number of agents or value of items.

cost. They will not tend to bid close to the item's value. In contrast, most relatively high probabilities are concentrated on the low portion of the price. For these reasons, the item's value has a weaker influence on probability distribution and the overall impact will not be significant. But it still causes slight decrease of $p(1)$.

According to Fig. 3, higher value and larger amounts of agents will make the curve wider and flatter with greater dispersion in price. Lower value and smaller amount of agents will lead to a steeper curve, where most agents bid on low prices. The number of agents, rather than the value of items, is the most significant factor in bid performing.

3.3. *Model fitting*

To test our model's predictability, we compare the predicted distribution (black solid line) with the actual bid frequency (gray histogram). In Figs. 4(a) and 4(b), the numbers of agents are set to be $N = 135$ and $N = 199$ respectively. In other two ones, numbers are much larger than 200 with $N = 945$ and $N = 2480$. From Fig. 4, we can draw the following conclusions:

(i) All four curves depict the decreasing tendency. The point whose probability is nearly to zero is far from item's value, which corresponds to the actual situation.

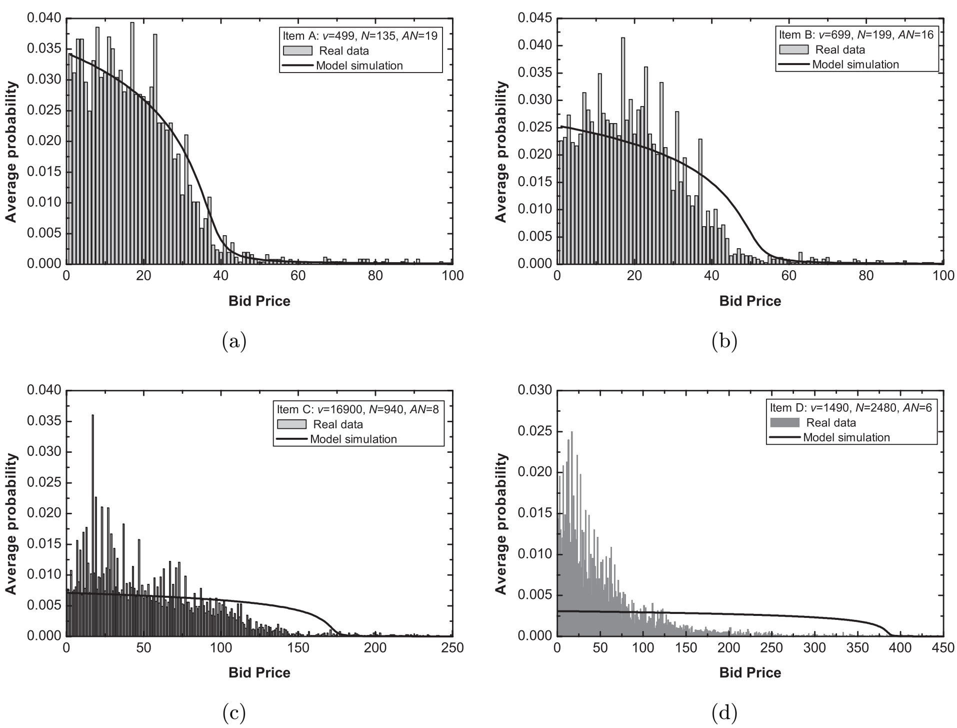


Fig. 4. Comparison between model and empirical data.

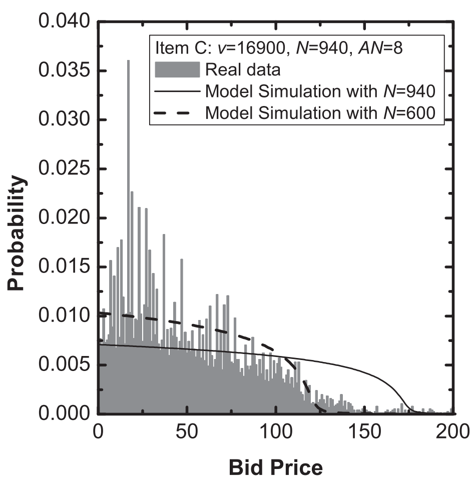
(ii) In the case of smaller numbers of bids N (Figs. 4(a) and 4(b)), the model fits data very well where all three regions are covered.

(iii) In the case of larger N (Figs. 4(c) and 4(d)), the fitness is far from satisfactory. This deviation can be reduced by bid number modification. We will address it in next section.

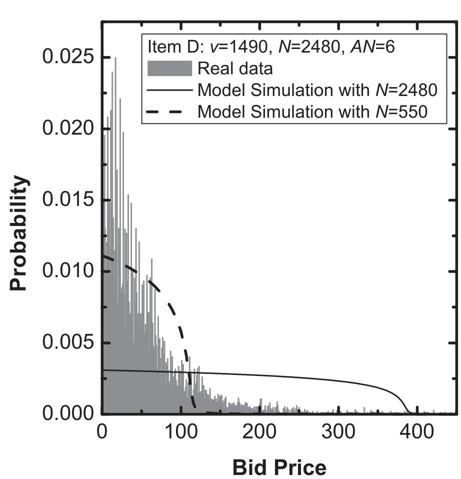
4. Cognitive Illusion and Correction in Bid Number

Our model's prediction in the case of small number of agents is good, but the fitness is not well for large number of agents. How can we improve it? In our opinion, it is because the inaccurate human thinking. Actually, in this kind of auction, agents will take price confliction and winning chance into consideration. This decision-making process is just a rough one without any strict mathematical calculations. So this kind of deviation in thinking is unavoidable, especially in difficult cases.

So, we speculate that the agents' estimation is more accurate for a smaller number of participants. For example, the model curves of items C and D in Fig. 4(b) are much flatter than the real data. According to our discussion of Sec. 3.2, if we change the bid numbers to smaller ones, these curves will become much steeper and fit data better (see Fig. 5 and Table 2).



(a)



(b)

Fig. 5. Comparison between two items with different N.

Table 2. Detailed information and KS test about two items.

Item	Bid number	Corrected bid number	KS	KS in corrected number
C	945	600	0.2315	0.0737
D	2480	550	0.5413	0.1174

It is obvious that the bid distribution derived from the model, represented by black solid line in Fig. 5, has a large deviation from the empirical data. Nevertheless, when we reduce the bid number to 600 and 550 respectively in the model, the resulted distribution given by the corresponding black dotted line has a smaller deviation than the former one. This can also be demonstrated by the Kolmogorov–Smirnov (KS) statistic test whose value after number correction is much smaller than the original one. Although the new fit is still not perfect, it sounds much better. As the agents are in larger quantities, they may not have exact information on the bid number and underestimate it. As a result, what they act in this case is just like that in the cases of smaller quantity.

5. Conclusion

Research on human behavior always plays an important role in science. In this paper, we make attempts to find a reasonable mechanism for the LUBA, a typical human behavior. First, we illustrate the bid probability distribution characters. We divide the curve into three regions and discuss the influence imposed by the number of agents and the value of items. Then, we develop a model parameterized by the above two deciding factors to explain the bid probability distribution. In the model, we assume that the agents are homogeneous, who allocate sources in proportion to opportunities. The model fits well in the case of small bid numbers. For a large N, the model has a flatter and wider plateau region than the actual situation. We attribute this deviation to the cognitive illusion which means that the human brain does not always carry out a very rigorous calculation for the reason of lacking information. While the number of agents is too big, their reasoning will have a big deviation from strict calculation. However, our model is only an attempt in the extensive study of human decision-making behavior and motivation. There are still many points needed to make clear.

Acknowledgments

The authors are grateful to Prof. Simone Pigolotti for sharing the data. This work was supported by the National Natural Science Foundation of China (NSFC) under the grant No. 61174165, the Program for New Century Excellent Talents in University (NCET-10-0245) and the Fundamental Research Funds for the Central Universities. Part of this work is also funded by China Scholarship Council.

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